Signal-to-Noise Ratio (of Detection)

- Arrivals are stochastic, Poisson
  \[ \mathbb{V}[\lambda] = \lambda \]

- Declared that our mean arrival rate is \( \bar{\lambda} \)

Raw image \( \text{SNR} = \frac{\text{mean}}{\text{var}} = \frac{\bar{\lambda}}{\sqrt{\bar{\lambda}}} = \sqrt{\bar{\lambda}} \)

\( \Rightarrow \) more photons = more SNR

On a pixel basis, we have \( \bar{\lambda} \) photons spread out over a grid (JxJ pixels). \( \text{SNR}_{\text{pixel}} = \frac{1}{\sqrt{J}} \left( \frac{\text{higher \, SNR}}{\text{lower \, SNR}} \right) \)
Contrast to Noise is what matters...

Contrast = fractional change in counts for a pixel relative to background.

\[ C = \frac{N_e - N_b}{N_b} \]

- \( \bar{N}_b = \text{mean background count [noise floor]} \)
- \( \bar{N}_e = \text{mean target count} \)
We don't know $\bar{N}_b$ or $\bar{N}_f$ a priori.

\[ \Rightarrow \text{Can estimate } \eta_b \bar{N}_b \]

Local $\text{SNR} = \text{SNR}_L = \frac{\bar{N}_b - \bar{N}_f}{\sqrt{\bar{N}_f}} = C \sqrt{\bar{N}_f}$

$C$ noise variance computed based on "non-target" pixels.

$\Rightarrow \text{SNR, CNR computed post-hoc, but can use this information to "tune" acquisition.}$

$\Rightarrow \text{Guess what is the target!}$
• Identify $N$ pixels likely w/in target
• Identify $M$ pixels likely w/in blkgd

\[ \frac{\hat{N}_t}{N_t} = \frac{1}{N} \sum_{n=1}^{N} t_n \leq \text{photon counts from "target" pixels} \]

\[ \frac{\hat{N}_b}{N_b} = \frac{1}{M} \sum_{m=1}^{M} b_m \leq \text{photon counts from "blkgd" pixels} \]

\[ \text{SNR}_{\hat{N}} = \frac{\frac{\Delta}{\hat{N}_t} - \frac{\Delta}{\hat{N}_b}}{\sqrt{\frac{\hat{N}_t}{\hat{N}_b}}} \quad \text{"more exposure \ldots time = better"} \]
Resolution in emission tomography

⇒ Detection is affected using "detector blocks".

- Example: 2x2 PMTs per crystal subdivision

- 9 PMTs backing a crystal segmented into 2x2 space.
Scoring limits spread within crystal

1. Generally known which crystal was excited

2. Use center mass to 1D slice location of event

- Define $(0,0)$ as center of crystal
- PMT is centered at $(x_k, y_k)$

Amplitude of current observed in the $k^{th}$ PMT = $a_k$ ["height"]

Total height: $z = \sum_k a_k$

$x_{\text{target}} = \frac{1}{z} \sum_k x_k a_k$

$y_{\text{target}} = \frac{1}{z} \sum_k y_k a_k$
Effect of collimation on resolution

Resolution due to collimator:

\[ R_C = \frac{d}{l} (d + b + R) \]

\[ \text{FWHM} \approx 20 \sqrt{\text{psf}} \]

- Resolution degrades with distance
- Further distance = less differential in angle... less resolupur
Planar camera (SPECT) PSF is approx as

$$h_c(x, y; R) = e^{-4(x^2+y^2)/R^2}$$

Assumes all photons striking voxels are absorbed.

Resolution as for \( f(R) \)

\( R_c \) (with \( E \))

\( R_c = \frac{d}{\mu} (R_c + L_R) \)

"Effective hole length": \( L_e = L - 2\mu \)

Not quite true...

\[ \mu \approx \frac{1}{e} \]

Notation useful for voxel material.
Intrinsic Resolution (SPECT)

\[ h_I(x,y) = e^{-A(x^2+y^2) \frac{\ln 2}{R^2_I}} \]
related to inherent accuracy loss
in the "camera"

\[ R_I = f[I_1, I_2] \]

\[ \Rightarrow [1] \text{ May get Compton scattering before absorption} \]
\[ \Rightarrow \text{get additional (small) light flashes} \]
\[ \Rightarrow \text{at some distance from the actual absorption} \]
\[ \Rightarrow \text{blur the center of mass calculation} \]
[2] = Scintillated photons are not well-behaved

⇒ it produced AND their spread are

both RANDOM!

amount & spread of light w/in PMTs

are also RANDOM!

⇒ e⁻ cascade in the PMT

is RANDOM!
So we have

\[ h_C \leftrightarrow \text{blue due to collimation [if present]} \]

\[ h_I \leftrightarrow \text{blue due to stochastic nature of detection} \]

Both \( h_C(x,y) \) and \( h_I(x,y) \) are convolved with our projection \( \phi_d \).

\[
(20s - 60s) \quad \rightarrow \text{Cannot readily be undone...}
\]

\[
\text{No simple \textit{filter}}
\]

\[ \text{Soln: Collect large \# of events... averaging over time} \]

\[ \text{Estimate B location \& height will be accurate \textit{on average}} \]